

```
function lhs = smooth_mklhs(kpts)
% The lhs of the system of equations smoothest forward curves.
%
% kpts: Time moments associated with the discount factor array.

% First, create the matrix: each knot point will add five more
% coefficients. Thus the size of the dense matrix that we are interested
% in is [5*length(kpts)]x[5*length(kpts)]. Of course, we use at most ten
% coefficients on any one line.

m = zeros(5 * length(kpts));

% Note that the first and the last block of equations is special.

% Set up the conditions at t = 0.

% fwd'''(0)
lhs(1, 2) = 6;

% fwd''(0)
lhs(2, 3) = 2;

% Base index for the current unknowns. The coefficients of the current
% interval have indices bindex..bindex+4, and the coefficients of the next
% one have indices bindex+5..bindex+9.
bindex = 1;

% Counter that points to the current equation (the one that is currently
% built).
eqn = 3;

% These are the powers of the "preceding" knot point. Recall that 0 is always
% the first (implicit) knot point.

tprec = 0;

% Now set up the blocks of five equations, one for each knot point,
% except for the last one.

for i = 1 : length(kpts) - 1

    % Compute the powers of kpts(i).
    t1 = kpts(i) - tprec;
    t2 = t1 * t1;
    t3 = t1 * t2;
    t4 = t1 * t3;
    t5 = t1 * t4;
```

```
% Continuity of the 3rd derivative at the right hand side of the interval;
;
% Continuity of the 2nd derivative at the right side of the interval;
% Continuity of the 1st derivative at the right side of the interval;
% Continuity of the curve at the right side of the interval;
% Condition on the price at the right end of the current interval.
lhs(eqn : eqn + 4, bindex : bindex + 9) = ...
[ ...
  24*t1   6           0   0   0   0 -6   0   0   0; ...
  12*t2   6*t1       2   0   0   0   0 -2   0   0; ...
  4*t3   3*t2   2*t1   1   0   0   0   0 -1   0; ...
  t4     t3   t2   t1   1   0   0   0   0 -1; ...
  t5/5   t4/4   t3/3 t2/2 t1   0   0   0   0   0 ...
];
;

% In the next cycle, this knot point will become the previous one.
tprec = kpts(i);

% Move to the next equation and to the next set of unknowns.
eqn = eqn + 5;
bindex = bindex + 5;

end

t1 = kpts(end) - tprec;
t2 = t1 * t1;
t3 = t1 * t2;
t4 = t1 * t3;
t5 = t1 * t4;

% Now add the equations that refer to the last interval.

% fwd'''(end)
tmp1 = [24*t1 6 0 0 0];

% fwd''(end)
tmp2 = [12*t2 6*t1 2 0 0];

lhs(eqn : eqn + 2, bindex : bindex + 4) = ...
[ ...
  t5/5 t4/4 t3/3 t2/2 t1; ... % Price at the right end of the interval.
  tmp1;
  tmp2
];
```